

Mark DeBree, Senior A/LM Analyst

STOCHASTIC VALUATION IN INTEREST RATE RISK MANAGEMENT

Southwest Corporate Investment Services (SCIS) has the ability to provide Stochastic Valuation, which is a more complex form of determining the value of an instrument by randomly generating interest rate paths. This methodology is used to calculate a more accurate market value of an instrument and is provided for our Asset/Liability Management Service clients that subscribe to the Comprehensive A/LM Service.

This is the first of a three part series on this subject that will: I) define the terms used in this type of analysis, II) illustrate how the SCIS A/LM Department utilizes this type of analysis, and III) provide two examples of the computation and results of this analysis.

Part I: Stochastic Valuation Terms

Antithetic Sampling – Similar to Monte Carlo sampling, but generates every other path in the lattice and creates an equal number of paths that are mirror images of the generated paths. Because the system generates paths in antithetic pairs, estimates of mean values converge more quickly to their true values than they with pure Monte Carlo simulation.

Arbitrage - a transaction or portfolio that makes a profit without risk. Arbitrage is also defined as a trading strategy that requires no investment of capital and will return a profit.

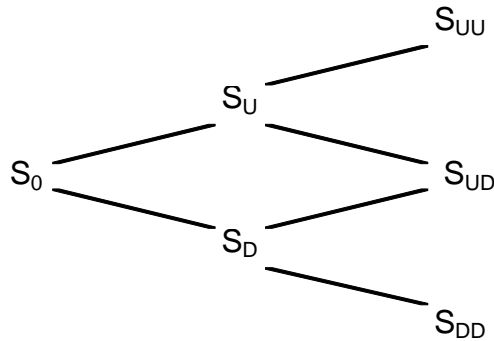
Arbitrage-free pricing – A market is said to have no arbitrage -- or be arbitrage free -- if prices in that market offer no arbitrage opportunities. This is a theoretical condition that is usually assumed for markets in economic and financial models.

Backward Induction – a technique that employs lattice-based valuation. This technique is most suitable for instruments that have embedded options with multiple exercise dates that are not path dependent, such as the call provision with callable securities. This valuation technique begins at the final maturity date and moves “backwards” through the lattice to determine a price or value “today.”

Binomial Model – a valuation model used to estimate the current price of an asset by evaluating how an asset’s price evolves over successive short periods of time. In each short period, it is assumed that only two price movements can occur (up or down, up or flat, flat or down).

Binomial lattice (tree) – a lattice or “tree” that represents how an asset’s price evolves over time within the Binomial Model. See Illustration A.

Binomial Tree
Illustration A



Where:

S_0 = Spot or current price

S_U = Price given One Up movement in interest rates

S_D = Price given One Down movement in interest rates

S_{UU} = Price given Two Up movement in interest rates

S_{DD} = Price given Two Down movement in interest rates

S_{UD} = Price given One Up and Down movement in interest rates

Black-Karasinski – a single factor term structure of interest rates model that attempts to define the future shape of the yield curve. Using this model, short term interest rates are assumed to follow log-normal distribution rather than a normal distribution, which prevents interest rates from becoming negative and allows interest rates to have large positive values.

Embedded Options – an option that is an inseparable part of another instrument such as the call provision on an Agency security or the mortgage prepayment option retained by borrowers.

Forward Interest Rates – a projection of future rates calculated from the current spot rates. Forward interest rates are considered to be an unbiased predictor of future interest rates, but empirically this theory does not hold.

Hull-White – a single factor term structure of interest rates model that attempts to define the future shape of the yield curve. This model assumes that short term interest rates follow a normal distribution, which allows interest rates to become negative, but also limits interest rates from becoming too large by applying a mean reversion.

Implied Volatility – a method of determining the amount of volatility that the financial markets are pricing into a derivative instrument given the market value and characteristics of the underlying asset. Implied volatilities are typically computed based on the various terms and option maturities for swap options and the underlying swaps.

Lattice based valuation - Lattice valuation can be used to value options with one or more exercise dates. Lattice valuation is performed in two different manners depending upon the option type: backward induction and path-wise simulation.

Lognormal distribution - the distribution of a random variable whose natural logarithm is normally distributed. A lognormal distribution is skewed so that a higher proportion of possible returns exceed the expected value versus falling short of the expected value. In the lognormal forecasting model, one plus the total return has a lognormal distribution.

Mean Reversion – a theory suggesting that a value will eventually move back towards its mean or average. The mean or average can be the historical average or another relevant average such as the industry average.

Monte Carlo sampling- the generation of sets of completely random numbers that are transformed into the surprise component of the path generated.

Multi-factor model - a model of financial instrument returns (or other variables) that accounts for the impact of multiple factors. These other factors include, but are not limited to return correlations between asset classes, past and current level of interest rates, volatilities, etc.

Net Economic Value (NEV) – estimates the theoretical market value of equity by taking the difference between the estimated market value of all earning assets and cost liabilities. NEV is a measure of the net benefit/detriment to the credit union that accrues from how each asset or liability is priced relative to market interest rates.

Normal Distribution – the normal distribution (a bell-shaped curve) represents a theoretical frequency distribution of measurements. In a normal distribution, scores are concentrated near the mean and decrease in frequency as the distance from the mean increases.

Path – a probable future path that interest rates can take given current interest rates, volatility, and a mean reversion if necessary (determined by the term structure model used Hull-White or Black-Karasinski).

Path-wise simulation – a specific valuation technique for instruments with path dependent options. The most common path dependent options are found in retail loans such as prepayments on residential home mortgages. To value an instrument using path-wise simulation with the highest degree of accuracy, it is necessary to simulate every path and compute the value as a probability weighted average across all the paths. This process is often not practical since the number of paths in a lattice grows exponentially with the size of the lattice. As a substitute for simulating every path, estimated market value is calculated by taking the average across a sample of possible paths. This sample is generated using Monte Carlo sampling or Antithetic sampling.

Simulation – an analytical method meant to imitate a real-life system. Common uses are to estimate income in the future, such as income simulation.

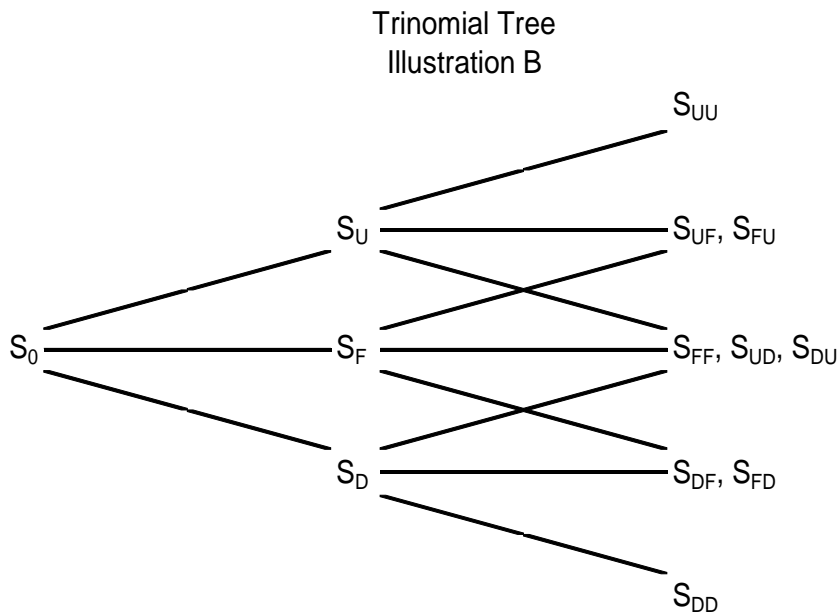
Single-factor model – a model of financial instrument returns that acknowledges only one common factor. The single factor is typically the market return.

Spot Rate – the current market rate for a given term or tenor.

Stochastic Simulation – a modeling technique that employs probability distributions rather than point estimates of the possible future trends to derive potential future outcomes. It allows for correlations between combinations of assumptions and can be used also to narrow the most likely future outcomes.

Term Structure of Interest Rates – also known as the *Yield Curve*. This is a curve that shows the relationship between bond yields and their respective maturity dates for a set of similar bonds, usually Treasuries, at a given point in time.

Trinomial lattice (tree) – an extension of the Binomial lattice technique that allows for a third possible rate movement. See Illustration B.



Where:

S_0 = Spot or current price

S_U = Price given One Up movement in interest rates

S_F = Price given One No Change in interest rates

S_D = Price given One Down movement in interest rates

S_{UU} = Price given Two Up movement in interest rates

S_{UF}, S_{FU} = Price given One Up movement and One No Change in interest rates

S_{FF}, S_{UD}, S_{DU} = Price given One Up and Down movement or Two No Changes in interest rates

S_{DF}, S_{FD} = Price given One Down movement and One No Change in interest rates

S_{DD} = Price given Two Down movement in interest rates



Special Report

May 11, 2006

Valuation – the process of determining an estimated value or worth of a financial instrument. There are many techniques that can be used: Two very common methods are discounted cash flows (DCF) and lattice-based valuations.

Volatility – a measure of the uncertainty or variability of a return realized on an asset.

Yield Curve – the relationship between bond yields and their respective maturity dates for a set of similar bonds, usually Treasuries, at a given point in time.

These definitions provide the foundation for a better understanding of stochastic valuation. Part II in this series will appear in June 2006 and will illustrate how the SCIS A/LM Department utilized this valuation method.

If you have any questions please contact:

Pat Gawenda, Manager – A/LM Modeling and Analysis, Gawendap@swcorp.org

Mark DeBree, Senior A/LM Analyst, DeBree@swcorp.org

Amy Lin, Senior A/LM Analyst, Lina@swcorp.org

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Investopedia: <http://www.investopedia.com>

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